

平行弦中點軌跡

錐線 $ax^2 + bxy + cy^2 + dx + ey + f = 0$ ，以 m 為斜率之諸平行線中點軌跡為

何？

$$\begin{cases} ax^2 + bxy + cy^2 + dx + ey + f = 0 \\ y = mx + k \end{cases}$$

$$ax^2 + bmx^2 + bmx + cm^2x^2 + 2ckmx + ck^2 + dx + emx + ek + f = 0$$

$$(a + bm + cm^2)x^2 + (bk + 2ckm + d + em)x + (ck^2 + ek + f) = 0$$

$$x = \frac{x_1 + x_2}{2} = \frac{-(bk + 2ckm + d + em)}{2(a + bm + cm^2)}$$

$$k = \frac{2(a + bm + cm^2)x + d + em}{-(b + 2cm)}$$

$$\text{故 } y = mx - \frac{(2a + 2bm + 2cm^2)x + d + em}{b + 2cm} = \frac{-2ax - bmx - d - em}{b + 2cm}$$

$$\text{即 } by + 2cmy + 2ax + bmx + d + em = 0 \Rightarrow \boxed{2ax + bmx + by + 2cmy + d + em = 0}$$

可視為

若將錐線方程式對 x 微分

$$\text{得 } 2ax + by + bxy' + 2cyy' + d + ey' = 0$$

$$\text{整理得 } y' = \frac{-(2ax + by + d)}{bx + 2cy + e}$$

另 $y' = m$ 即得

$$\boxed{(2ax + by + d) + m(bx + 2cy + e) = 0}$$

註：其必過中心