拉格朗日插值公式

例 1：已知 $f(x)$ 爲一 3 次多項式，且 $f(0)=-16, f(1)=2, f(2)=3, f(3)=1$，試求 $f(x)$？

解一：設 $f(x)=ax^3+bx^2+cx+d$，將 $f(0)=-16, f(1)=2, f(2)=3, f(3)=1$ 代入，可得四個聯立方程組，四個聯立式解 4 個未知數 a,b,c,d，故可解得 $f(x)$

解二：設 $f(x)=(-16)\dfrac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}$

$$+(2)\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}$$

$$+(3)\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}$$

$$+(1)\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$=\frac{8(x-1)(x-2)(x-3)}{3}-x(x-2)(x-3)-\frac{3x(x-1)(x-3)}{2}+\frac{x(x-1)(x-2)}{6}$$

$$=\frac{1}{3}x^3-\frac{11}{2}x^2+\frac{115}{6}x-16$$

練習 1：已知 $f(x)$ 爲一 2 次多項式，且 $f(1)=2, f(2)=3, f(3)=1$，試求 $f(x)$？

練習 2：已知 $f(x)$ 爲一 3 次多項式，且 $f(0)=-1, f(1)=1, f(2)=2, f(3)=3$，試求 $f(x)$？

拉格朗日插值公式

n 次多項式 $f(x)$，若 $f(x_0)=y_0$，$f(x_1)=y_1$，……，$f(x_n)=y_n$，則 $f(x)=$

$$y_0\times\frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)}+y_1\times\frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)}+$$

$$y_2\times\frac{(x-x_0)(x-x_1)...(x-x_n)}{(x_2-x_0)(x_2-x_1)...(x_2-x_n)}+......+y_n\times\frac{(x-x_1)(x-x_2)...(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)...(x_n-x_{n-1})}$$

證明：將 x_0 代入，則 $f(x_0)=y_0$ 正確

將 x_1 代入，則 $f(x_1)=y_1$ 正確

……

將 x_n 代入，則 $f(x_n)=y_n$ 正確

故 $f(x)=$

$$y_0\times\frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)}+y_1\times\frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)}+$$

$$y_2\times\frac{(x-x_0)(x-x_1)...(x-x_n)}{(x_2-x_0)(x_2-x_1)...(x_2-x_n)}+......+y_n\times\frac{(x-x_1)(x-x_2)...(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)...(x_n-x_{n-1})}$$

滿足題目所給各項條件 $f(x_0)=y_0$，$f(x_1)=y_1$，……，$f(x_n)=y_n$